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A Unitary Symmetry Model for Photoproduction.

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In this note we would like to consider photoproduction of pseudoscalar mesons from the viewpoint of unitary symmetry. The underlying basis of these considerations is the assignment of the known (or conjuctured) particles and resonances to irreducible representations of a group which satisfies the Lie Algebra. Once this assignment has been made the problem is reduced, as far as the group-theoretical part is concerned, to finding the irreducible representations which are contained in the direct-product space of two representations. Results are then obtained in the form of relations between the amplitudes for various photoptoduction processes.

Of the four simple compact Lie groups of rank two, the one called SU_3 lends itself to simple correspondences with the known particles namely the Sakata model and the a eight-fold way of GELL-MANN (1) and NE'MAN (2). The Sakata model is based on the following assignment:

$$D^3(1, 0) \leftrightarrow (pn\Lambda) ,$$

$$D^8(1, 1) \leftrightarrow (\pi K\eta) ,$$

and the eight-fold way is based on the following assignment:

$$D^8(1, 1) \leftrightarrow (\mathcal{N}\Lambda\Sigma\Xi) ,$$

$$D^8(1, 1) \leftrightarrow (\pi K\eta) ,$$

$D^n(\lambda_1, \lambda_2)$ denotes an irreducible representation of SU_3 where n is the dimensionality of the representation and the highest weight is $\lambda_1 m_1+\lambda_2 m_2$ where m_1 and m_2 are the two fundamental dominant weights of the group (3).

The two mutually commuting operators of the group, H_1 and H_2, are identified with hypercharge Y and the third component of isotopic spin T_3, respectively.

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(2) Y. NE'MANN: *Nucl. Phys.*, **26**, 222 (1961).
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From the theory of Lie algebras it is known that one can write the commutation relations of the group generators in the form

$$[F_m, F_n] = if_{mnl}F_l ,$$

where the f's are real and completely antisymmetric. The charge Q can be written as

$$Q = T_3 + Y/2 .$$

For the eight-fold way, where one makes the following correspondences:

$$H_1 = F_3 = T_3 ,$$

and

$$2H_2 = (2/\sqrt{3})F_8 = Y ,$$

the electromagnetic current will be given by (4)

$$j_{\text{em}} = j_3 + (1/\sqrt{3})j_8 .$$

The currents j_m satisfy

$$[F_m, j_n] = if_{mnl}\,j_l ,$$

i.e., they belong to the regular eight-dimensional representation.

We first reduce the $|BM\rangle$ states, where B is a baryon and M a meson, into a eigenstates of the $8\otimes 8$ irreducible product representations.

We write

$$|BM\rangle = \sum_{iT} |n_i;\, YT\rangle\, a_{iT} ,$$

where in the state vector $|n_i,\, YT\rangle$, n corresponds to the dimensionality of the representation and Y and T are the hypercharge and isotopic spin respectively. Furthermore we need the transformation properties of $\langle\mathcal{N}|j_{\text{em}}$ where $\mathcal{N}$ is a nucleon:

$$\langle\mathcal{N}|j_{\text{em}} \sim \sum_{iT} \langle n_i;\, YT|\, b_{iT} .$$

The generalized Clebsch-Gordan coefficients a_{iT} and b_{iT} can be obtained from the equations for the product space state vectors (5). The photoproduction amplitudes can then be written as

$$\langle\mathcal{N}|j|BM\rangle = \sum_{iT} A_i b_{iT} a_{iT} ,$$

(see Table I). The A's are the group amplitudes and are eight in number; $A_1 A_8 A_{8'} A_{10} A_{\overline{10}} A_{27} A_{8'8}$ and $A_{88'}$. The amplitudes $A_{8'8}$ and $A_{88'}$ exist because of the

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TABLE I.

	A_{27}	$A_{\overline{10}}$	A_{10}	A_8	$A_{8'}$	$A_{88'}$	$A_{8'8}$
$\langle p\lvert j\rvert p\pi^0\rangle$	$2/5$		$1/3$	$1/10$	$1/6$	$-1/(6\sqrt{5})$	$-1/(2\sqrt{5})$
$\langle p\lvert j\rvert n\pi^+\rangle$	$\sqrt{2}/10$		$\sqrt{2}/6$	$-\sqrt{2}/10$	$-\sqrt{2}/6$	$1/(3\sqrt{10})$	$1/\sqrt{10}$
$\langle p\lvert j\rvert p\eta\rangle$	$\sqrt{3}/5$			$-1/(10\sqrt{3})$	$\sqrt{3}/6$	$-1/(2\sqrt{15})$	$1/(2\sqrt{15})$
$\langle p\lvert j\rvert \Sigma^+ K^0\rangle$	$\sqrt{2}/10$		$-\sqrt{2}/6$	$-\sqrt{2}/10$	$\sqrt{2}/6$	$-1/(3\sqrt{10})$	$1/\sqrt{10}$
$\langle p\lvert j\rvert \Sigma^0 K^+\rangle$	$2/5$		$-1/3$	$1/10$	$-1/6$	$1/(6\sqrt{5})$	$-1/(2\sqrt{5})$
$\langle p\lvert j\rvert \Lambda K^+\rangle$	$\sqrt{3}/5$			$-1/(10\sqrt{3})$	$-\sqrt{3}/6$	$1/(2\sqrt{15})$	$1/(2\sqrt{15})$
$\langle n\lvert j\rvert n\pi^0\rangle$	$3/10$	$1/6$	$1/3$	$1/5$			
$\langle n\lvert j\rvert p\pi^-\rangle$	$\sqrt{2}/5$	$-\sqrt{2}/6$	$\sqrt{2}/6$	$-\sqrt{2}/5$			
$\langle n\lvert j\rvert n\eta\rangle$	$\sqrt{3}/10$	$\sqrt{3}/6$		$\sqrt{3}/15$			
$\langle n\lvert j\rvert \Sigma^0 K^0\rangle$	$3/10$	$-1/6$	$-1/3$	$1/5$			
$\langle n\lvert j\rvert \Sigma^- K^+\rangle$	$\sqrt{2}/5$	$\sqrt{2}/6$	$-\sqrt{2}/6$	$-\sqrt{2}/5$			
$\langle n\lvert j\rvert \Lambda K^0\rangle$	$\sqrt{3}/10$	$-\sqrt{3}/6$		$\sqrt{3}/15$			

equivalence between the 8 and 8′ representations in the direct product

$$8\otimes 8 = 1 + 8 + 8' + 10 + \overline{10} + 27\,.$$

The 10 and $\overline{10}$ representations are inequivalent.

The following relations can then be written for the photoproduction amplitudes [the symbol $A(\mathrm{n}\pi^+)$ denotes $A(\gamma+\mathrm{p}\to\mathrm{n}+\pi^+)$ etc.]:

(1) $$\sqrt{2}\,A(\mathrm{n}\pi^+) + A(\Sigma^0\mathrm{K}^+) = \sqrt{3}\,A(\Lambda\mathrm{K}^+)\,,$$

(2) $$\sqrt{2}\,A(\Sigma^+\mathrm{K}^0) + A(\mathrm{p}\pi^0) = \sqrt{3}\,A(\mathrm{p}\eta)\,,$$

(3) $$A(\mathrm{n}\pi^0) - \sqrt{3}\,A(\mathrm{n}\eta) = \sqrt{3}\,A(\Lambda\mathrm{K}^0) - A(\Sigma^0\mathrm{K}^0)\,,$$

(4) $$\sqrt{2}\,A(\Sigma^0\mathrm{K}^0) + A(\Sigma^-\mathrm{K}^+) = \sqrt{2}\,A(\Sigma^0\mathrm{K}^+) + A(\Sigma^+\mathrm{K}^0)\,,$$

(5) $$\sqrt{2}\,A(\mathrm{n}\pi^0) + A(\mathrm{p}\pi^-) = \sqrt{2}\,A(\mathrm{p}\pi^0) + A(\mathrm{n}\pi^+)\,.$$

The relations (1) and (2) yield the following triangle inequalities for the cross-sections:

$$2\sigma(\mathrm{n}\pi^+) \leqslant \sigma(\Sigma^0\mathrm{K}^+) + 3\sigma(\Lambda\mathrm{K}^+)\,,$$

$$\sigma(\mathrm{p}\pi^0) \leqslant 2\sigma(\Sigma^+\mathrm{K}^0) = 3\sigma(\mathrm{p}\eta)\,.$$

One can, however, obtain stronger predictions if one of the group amplitudes becomes dominant, *e.g.* through a resonance. For instance the $J=\frac{3}{2}$, $T=\frac{3}{2}$ resonance in the pion-nucleon system can be considered as a member of $D^{10}(1, 0)$.

Then with A_{10} dominant the cross-sections at resonance become

$$\sigma(\mathrm{p}\pi^0) = 2\sigma(\mathrm{n}\pi^+) = 2\sigma(\Sigma^+\mathrm{K}^0) = \sigma(\Sigma^0\mathrm{K}^+)\,,$$

and

$$\sigma(\mathrm{p}\eta) = \sigma(\Lambda\mathrm{K}^+) = 0\,.$$

This model fails, at least for the ΣK part, because the $\frac{3}{2}$, $\frac{3}{2}$ resonance is below the physical threshold.

If the A_8 amplitude becomes dominant, for instance at one of the higher nucleon resonances, then the following relations can be written:

$$\sigma(\mathrm{p}\pi^0) = \tfrac{1}{2}\sigma(\mathrm{n}\pi^+) = 3\sigma(\mathrm{p}\eta) = \tfrac{1}{2}\sigma(\Sigma^+\mathrm{K}^0) = \sigma(\Sigma^0\mathrm{K}^+) = 3\sigma(\Lambda\mathrm{K}^+)\,.$$

We also observe that if $A_{88'}$ and $A_{8'8}$ are zero and all the others are equal with the same phase, then $\sigma(\mathrm{p}\pi^0) = \sigma(\mathrm{n}\pi^0)$ and all other cross-sections are zero.

Let us now find the corresponding relations between the photoproduction amplitudes for the Sakata model. If we note that the phenomenological interaction Lagrangian must be invariant under rotations in the unitary space then it must be of the form

$$\mathscr{L}' \sim a_1\,\mathrm{Tr}\,[\bar{B}\bar{M}QB] + a_2\,\mathrm{Tr}\,[\bar{B}Q\bar{M}B] + a_3\,\mathrm{Tr}\,[\bar{B}B]\,\mathrm{Tr}\,[Q\bar{M}]\,,$$

where

$$B = \begin{pmatrix} \mathrm{p} \\ \mathrm{n} \\ \Lambda \end{pmatrix},$$

$$M = \begin{pmatrix} (1/\sqrt{6})\eta + (1/\sqrt{2})\,\pi^0 & \pi^+ & K^+ \\ \pi^- & (1/\sqrt{6})\,\eta - (1/\sqrt{2})\,\pi^0 & K^0 \\ K^- & \overline{K^0} & -(2/\sqrt{6})\,\eta \end{pmatrix},$$

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The a's in the above equation are linearly related to the irreducible group-amplitudes. Performing the indicated traces enables us to obtain the following relations for the photoproduction amplitudes:

$$\sqrt{3}\,A(\mathrm{p}\eta) = A(\mathrm{p}\pi^0)\,,$$

$$A(\mathrm{n}\pi^+) = A(\Lambda\mathrm{K}^+)\,,$$

$$A(\mathrm{n}\pi^+) + A(\mathrm{p}\pi^-) = \sqrt{2}\,A(\mathrm{p}\pi^0) - \sqrt{2}\,A(\mathrm{n}\pi^0)\,,$$

$$\sqrt{3}\,A(\mathrm{n}\eta) = A(\mathrm{n}\pi^0)\,,$$

$$A(\Lambda\mathrm{K}^0) = 0\,.$$

* * *

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